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Book Review

Fractals in Science by A. Bunde and S. Havlin, Editors

Springer-Verlag, Berlin, 1994. 298 pages + IBM or Macintosh Diskette. $59.00

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Fractals have captured the imagination of scientists and nonscientists alike. A fractal is an object composed of ever smaller pieces that are copies of itself. As a nonfractal object is magnified beyond a certain characteristic scale, no new features are revealed. The length, surface area, or volume all have unique values when measured at this or higher resolution. However, as a fractal object is magnified, ever new features are revealed. When the fractal is measured at finer resolution, these additional features will be included and they will change the values measured for the length, surface area, or volume. Thus, the values measured for these properties depend on the resolution used to perform the measurement. An important lesson here is that the value of a property measured at one resolution is not a meaningful measurement of a fractal object. The meaningful measurement is to determine how a property depends on the resolution used to make the measurement.

Fractals are not limited just to objects but also include time series that have the same statistical properties as fractal objects. At first, these statistical properties may seem strange. However, they have been studied by mathematicians over the last 250 years. They only seem strange because the training of most scientists is limited to Gaussian distributions. For example, the moments of a fractal distribution may not exist. That is, as we analyze ever more data, the means of these samples may continually decrease or increase rather than approach a nonzero, finite limiting value. Consider a fractal time series with an infinite hierarchy of bursts within bursts of activity. As we use ever larger windows to determine the mean we capture ever more spaces between the bursts and thus the mean value of the time series tends toward zero.

Fractals have provided new fruitful areas of investigation in mathematics and beautiful computer art pictures. Fractals have been widely applied in physics and chemistry because they can capture the structure of complex objects, putting oversimplifications, such as the spherical cow, out to pasture. These methods hold great promise for biology where such complexity is the rule. However, until recently, the biophysical community has not been much aware of the value of fractals. But that is now changing. The number of papers on fractals in the biomedical literature is growing exponentially.

The use of fractals is now far too extensive to be covered by any one book. The book *Fractals in Science* edited by Armin Bunde and Shlomo Havlin provides a useful sampling of fractals primarily from physics and chemistry, with a seasoning of biological examples. It consists of independent articles written by authors who are leaders in their fields. These articles are clear and careful. Some derivations are terse, as is typical of such reviews rather than textbook expositions. The mathematical level is calculus and differential equations. (Those who want to see the metric topology needed for the technically pure definitions of fractal dimension could look at the books *Fractals Everywhere* by Barnsley or *Measure, Topology, and Fractal Geometry* by Edgar.)

The first chapter by Bunde and Havlin provides an overview of fractal definitions and properties for the nonexpert. Bak and Creutz describe how local adjustments in a system driven far from equilibrium by the input of energy can produce fractal patterns in time and space. Their self-organizing critical (SOC) system approach with many degrees of freedom contrasts with the concept of "chaos" which has a small number of degrees of freedom. They cover systems of spins, sand, earthquakes, $1/f$ noise, although they do not include one of my personal favorite data sets of the biggest sandpiles measured—rock volumes of avalanches in the Himalayas (*Phys. Rev.* E47:724, 1993). Buldyrev et al. cover fractals in biology and medicine, including: airways in the lung, dendrites of neurons, patterns formed by fluids and random walkers that may explain the mechanisms at work in similar biological structures, and the fractal dynamics of ion channels. They concentrate on the use of the variance as function of window size to test for long term correlations in the base-pair sequences in DNA. Understanding such fractal correlations may help to differentiate coding from noncoding regions and provide insights into the evolution of DNA. They also show how similar methods can tell the difference between the timing of the heartbeats in normal and sick hearts. Kertész and Vicsek describe the growth of surfaces and the physics of how they become smoother or rougher. Weiss, well known for his expertise in random walks, reviews both elementary and advanced properties of such excursions. The article is very comprehensive, but I wish he would have extended the results to include the language and methods of analysis of fractional Brownian motion. Daoud describes the fractal structure of polymers and their motion. Redner and Leyvraz describe the patterns and reaction kinetics of diffusion-limited reactions. Local fluctuations in concentration grow until the reactions slow because all of one species is used up in a local area so that the reaction can now only proceed on the lower dimensional boundary where all the species in the reaction are present. Avnir et al. describe how the rates of reaction depend on the accessible surface area for adsorption and the reactive area of the reaction itself. In Table 8.1, they list the reaction dimension for 49 reactions including reactions with and without a catalyst.

The book comes with an IBM or Macintosh diskette by Rapaport and Meyer. These illustrate ideas in the book with programs that compute models of fractal growth, a fluid of soft disks, polymer reptation, percolation, Ising spins, a SOC sandpile, and images of fractals. Chaos, in the form of the logistic map, and cellular automata are also included. The Macintosh version makes good use of the Mac interface, did not bomb, and ran reasonably fast on my IIfx. (Those interested in programs to use fractal methods to actually analyze their own data could look at the Pascal code to analyze time series in *Fractals: A User's Guide for the Natural Sciences* by Hastings and Sugihara.)

Additional information specifically about fractals in biophysics can be found in the collections of articles in the *Annals of Biomedical Engineering* (Vol. 18, No. 2, 1990) and the *IEEE Engineering in Medicine and Biology Magazine* (vol. 11, No. 2, June 1992) and in a new textbook (*Fractal Physiology* by Bassingthwaighte, Liebovitch, and West).

In summary, *Fractals in Science* shows how fractals have provided new insight into problems in physics, chemistry, and biology. It provides a good sampling of this field for the neophyte and a good review for the expert.